$$\int_0^2 e^{-x/2}\,dx$$

Section 8.4
Improper Integrals

Improper Integrals with Infinite Integration Limits

Integrals with infinite limits of integration are improper integrals...

1.) If f(x) is continuous on $[a,\infty)$, then

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^{b} f(x)\,dx.$$

2.) If f(x) is continuous on (-∞, b], then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_{a}^{b} f(x)\,dx.$$

3.) If f(x) is continuous on (-∞,∞), then

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

where c is any real number.

Parts 1 and 2

If the limit is finite the improper integral CONVERGES and the limit is the value of the improper integral

If the limit fails to exist, the improper integral DIVERGES

Part 3

The integral on the left-hand side of the equation CONVERGES if both improper integrals on the right-hand side converge, otherwise it DIVERGES and has no value.

$$\int_0^\infty e^{-x/2}\, dx =$$

$$\int_{1}^{\infty} \frac{1}{x}\, dx =$$

$$\int_1^{\infty} \frac{1}{x^2}\, dx =$$

Express the improper integral

$$\int_{-\infty}^{\infty} e^{x}\,dx$$

in terms of limits of definite integrals and then evaluate the integral.

Does the improper integral

$$\int_1^{\infty} \frac{dx}{x}$$

converge or diverge?

Evaluate

$$\int_0^\infty \frac{2\,dx}{x^2 + 4x + 3}$$ **or state that it diverges**

Evaluate

$$\int_{1}^{\infty} xe^{-x}\,dx \quad \text{or state that it diverges}$$

Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$$

Rotate $y = 1/x$ from $x = 1$ to ∞ around the x-axis. Find the volume.

$$\int_{1}^{\infty} \frac{1}{x^p}\,dx$$

$$\int_0^{\infty} \frac{2\,dx}{x^2 + 4x + 3} =$$

Find the volume when we rotate $y = xe^{-x}$ from $[0,\infty)$ around the x-axis.

$$\int_0^5 \frac{3}{x-2}\, dx =$$

Find p for which $\displaystyle\int_0^1 \frac{dx}{x^{p+1}}$ **converges**

$$\lim_{a\to 0}\int_a^1 \frac{1}{x^{p+1}}\,dx = \lim_{a\to 0}\int_a^1 x^{-p-1}$$

$$= \lim_{a\to 0} \left.\frac{x^{-p-1+1}}{-p-1+1}\right]_a^1 = \lim_{a\to 0} \left.\frac{x^{-p}}{-p}\right]_a^1$$

$$\int_1^5 \frac{3\,dx}{x-3} = \int_1^3 \frac{3}{x-3}\,dx + \int_3^5 \frac{3}{x-3}\,dx$$

$$= \lim_{a\to 3} \int_1^a \frac{3}{x-3}\,dx + \lim_{b\to 3} \int_b^5 \frac{3}{x-3}\,dx$$

$$= \lim_{a\to 3} 3\ln(|x-3|)\Big]_1^a + \lim_{b\to 3} 3\ln|x-3|\Big]_b^5$$

$$= \lim_{a\to 3} \left[3\ln(a-3) - 3\ln 2\right] + \lim_{b\to 3} \left[3\ln 2 - 3\ln(b-3)\right]$$

Diverges.

Improper Integrals with Infinite Discontinuities

Integrals of functions that become infinite at a point within the interval of integratino are improper integrals.

1.) If f(x) is continuous on (a,b], then

$$\int_a^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)\,dx$$

2.) If f(x) is continuous on [a,b), then

$$\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx$$

3.) If f(x) is continuous on [a,c] U (c,b], then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_0^3 \frac{dx}{(x-1)^{2/3}} = \int_0^1 \frac{1}{(x-1)^{\frac{2}{3}}}\,dx + \int_1^3 \frac{1}{(x-1)^{\frac{2}{3}}}$$

Evaluate

$$\int_{1}^{2} \frac{dx}{x-2}$$

Test for Convergence or Divergence

Comparison Test

Let f and g be continuous on $[a,\infty)$ with $0 \leq f(x) \leq g(x)$ for all $x \geq a$. Then

1.) $\int_a^\infty f(x)\,dx$ converges if $\int_a^\infty g(x)\,dx$ converges

2.) $\int_a^\infty g(x)\,dx$ diverges if $\int_a^\infty f(x)\,dx$ diverges.

Does the integral $\int_{1}^{\infty} e^{-x^2}\,dx$ converge?

$$\int_{-\infty}^{\infty} e^x \, dx =$$

$$\int_{-\infty}^{\infty} \frac{1}{1 + x^2}\, dx =$$